LESSON 33

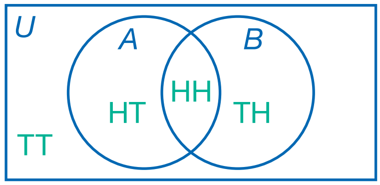
# Calculating Probability

## Independent and Dependent Events

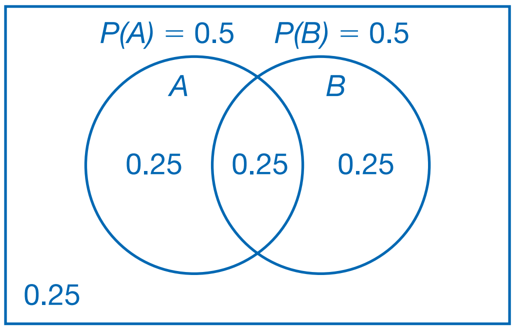
**UNDERSTAND** Sometimes, two or more events happen together. This is called a **compound event**. There are two types of compound events.

**Independent events** are events in which the outcome of one event does not affect the outcome of other events. For example, the outcome of tossing one coin does not affect the outcome of a second coin toss.

For **dependent events**, the outcome of one event affects the outcome of other events. An example is picking a marble from a bag containing marbles of different colors and then picking a second marble from the bag without replacing the first one. Whatever marble was picked first affects the probabilities of the second pick.




**UNDERSTAND** The probability that two events $A$ and $B$ will occur at the same time can be represented as the intersection of sets $A$ and $B$. Suppose two fair coins are tossed. Let's say that event $A$ is the first coin landing on heads, and event $B$ is the second coin landing on heads. This Venn diagram represents the possible outcomes and shows that $A \cap B = \{HH\}$.




A second Venn diagram can be created to represent the probabilities associated with tossing these two coins, as shown.

**Multiplication Rule for Independent Events:** The probability of the intersection of two subsets composed of independent events is equal to the product of their probabilities:

$P(A \cap B) = P(A)P(B)$

Substitute the probabilities into the formula to show that events $A$ and $B$ are independent.

$P(A \cap B) = P(A)P(B)$

$0.25 \stackrel{?}{=} (0.5)(0.5)$

$0.25 = 0.25$ ✓

If two events are independent, you can multiply their probabilities to find the probability that both will occur. This formula can also be used to determine if events are independent. If the probability of the events happening together or one after the other is equal to the product of their probabilities, the events must be independent.

# Connect

Holden will select marbles from two bags without looking. First, he will choose a marble from a bag that has 1 red marble and 2 blue marbles in it. Next, he will choose a marble from a different bag that has 4 red marbles, 2 blue marbles, and 2 green marbles in it. What is the probability that he will select a blue marble followed by a red marble?

**1** Determine the probabilities for each pick.

The first bag has 1 red marble and 2 blue marbles, so the probabilities for the first pick are:

$P(\text{red}) = \frac{1}{1+2} = \frac{1}{3}$

$P(\text{blue}) = \frac{2}{1+2} = \frac{2}{3}$

The second bag has 4 red marbles, 2 blue marbles, and 2 green marbles, so the probabilities for the second pick are:

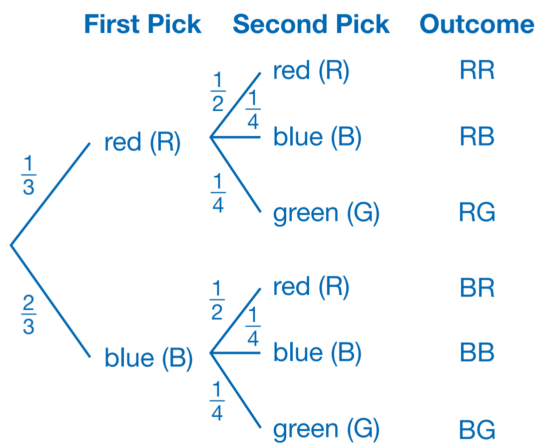
$P(\text{red}) = \frac{4}{4+2+2} = \frac{4}{8} = \frac{1}{2}$

$P(\text{blue}) = \frac{2}{4+2+2} = \frac{2}{8} = \frac{1}{4}$

$P(\text{green}) = \frac{2}{4+2+2} = \frac{2}{8} = \frac{1}{4}$

**2** Create a tree diagram, writing probabilities on the appropriate branches.

The first branch shows the first pick. The second branch shows the second pick.




**3** Determine the probability of choosing a blue marble followed by a red marble (BR).

The marbles are from different bags. The marble chosen from one bag does not affect the marble chosen from the other bag. So, the events are independent. Multiply the probabilities.

▶ $P(\text{BR}) = P(\text{blue first}) \cdot P(\text{red second})$

$= \frac{2}{3} \cdot \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$

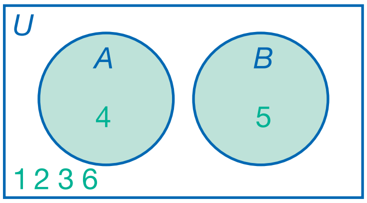
**DISCUSS**

The tree diagram shows a sample space with 6 possible outcomes. Why isn't $P(\text{BR})$ equal to $\frac{1}{6}$?

## The Addition Rule

**UNDERSTAND** If two events cannot happen together, they are **mutually exclusive events**. The probability of either of two mutually exclusive events occurring is equal to the sum of their individual probabilities. For example, suppose you toss a standard number cube. The result could be a 4, and the result could be a 5, but the result cannot be both a 4 and a 5 at the same time, as shown by the Venn diagram below. They are mutually exclusive events. So, the probability of tossing a 4 or a 5 is:

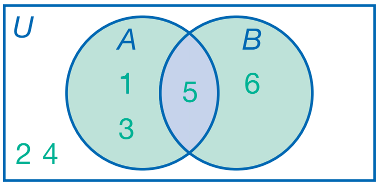
$P(4 \text{ or } 5) = P(4) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$




Events that are not mutually exclusive may have outcomes in common. For example, suppose you toss a number cube. Consider the event of a toss resulting in an odd number and the event of a toss resulting in a number greater than 4. Those events overlap, because 5 is a member of both sets, as shown by the Venn diagram below. So, to find the probability of tossing an odd number or a number greater than 4, apply the Addition Rule.

**Addition Rule:** The probability of the union of two subsets is equal to the sum of their individual probabilities minus the probability of the intersection of the subsets.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$




$P(\text{odd or} > 4) = P(\text{odd}) + P(>4) - P(\text{odd and} > 4) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

Consider the events as sums of the probabilities of mutually exclusive events.

$P(\text{odd}) = P(1 \text{ or } 3 \text{ or } 5) = P(1) + P(3) + P(5)$

$P(>4) = P(5 \text{ or } 6) = P(5) + P(6)$

$P(\text{odd and} > 4) = P(5)$

$P(\text{odd}) + P(>4) - P(\text{odd and} > 4) = P(1) + P(3) + P(5) + P(5) + P(6) - P(5)$

$P(\text{odd}) + P(>4) - P(\text{odd and} > 4) = P(1) + P(3) + P(5) + P(6) = P(\text{odd or} > 4)$

# Connect

The two-way frequency table shows the results of a survey of 11th-grade and 12th-grade students at Brookdale High School. Students were asked if they had a job last summer.

| | **Had Summer Job ($S$)** | **No Summer Job ($N$)** |
|---|---|---|
| **11th Grade ($E$)** | 18 | 22 |
| **12th Grade ($T$)** | 30 | 10 |

Use the Addition Rule to find $P(S \cup T)$. What does this probability represent?

**1** Write the Addition Rule, using sets $S$ and $T$.

$P(S \cup T) = P(S) + P(T) - P(S \cap T)$

**2** Find the probabilities for the sets and their intersection.

The total number of students surveyed was: $18 + 22 + 30 + 10 = 80$

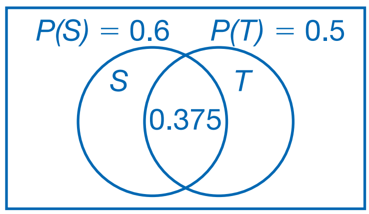
Consider the $S$ column and the $T$ row.

$P(S) = \frac{18 + 30}{80} = \frac{48}{80} = 0.6$

$P(T) = \frac{30 + 10}{80} = \frac{40}{80} = 0.5$

$P(S \cap T) = \frac{30}{80} = 0.375$

This could be represented in a Venn diagram as follows:




**3** Substitute the probabilities into the Addition Rule.

$P(S \cup T) = P(S) + P(T) - P(S \cap T)$

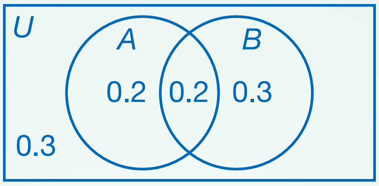
▶ $P(S \cup T) = 0.6 + 0.5 - 0.375 = 0.725$

This means that if you chose a student at random from a list of 11th- and 12th-grade students, you would have a 72.5% chance of selecting a student who is either a 12th-grade student or who had a summer job (or both).

**TRY**

Use the Addition Rule to find $P(N \cup E)$.

**EXAMPLE A** This Venn diagram shows the probabilities for events $A$ and $B$, their intersection, and the complement of their union. Are events $A$ and $B$ independent events?




**1** Choose a rule to use to determine if events $A$ and $B$ are independent.

If $A$ and $B$ are independent events, then $P(A)P(B) = P(A \cap B)$.

**2** Identify or determine the necessary probabilities.

The overlap shows the intersection of $A$ and $B$.

So, $P(A \cap B) = 0.2$.

$P(A)$ and $P(B)$ refer to the whole circles, including the intersection.

Add together the probabilities within circle $A$.

$P(A) = 0.2 + 0.2 = 0.4$

Add together the probabilities within circle $B$.

$P(B) = 0.2 + 0.3 = 0.5$

**3** Apply the Multiplication Rule for Independent Events.

$P(A)P(B) \stackrel{?}{=} P(A \cap B)$

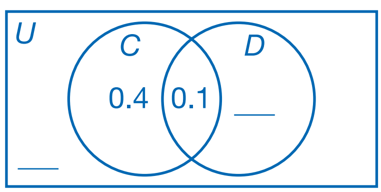
$(0.5)(0.4) \stackrel{?}{=} 0.2$

$0.2 = 0.2$ ✓

▶ The product of $P(A)P(B)$ is equal to $P(A \cap B)$, so events $A$ and $B$ are independent events.

**TRY**

Events $C$ and $D$ in the Venn diagram are independent events. Write the missing probabilities in the blanks of the Venn diagram.



**EXAMPLE B** An optional workshop to improve players' basketball skills was held for varsity players in the county. One month after the workshop, the organization that conducted it collected data about players who did and did not attend the workshop to see if any had improved their field goal percentages during games. The data are recorded in the table. Determine if attending the workshop and improving field goal percentages are independent events.

| | Attended | Did Not Attend | Total |
|---|---|---|---|
| **Improved** | 44 | 6 | 50 |
| **Did Not Improve** | 18 | 32 | 50 |
| **Total** | 62 | 38 | 100 |

**1** Find the probability that a local player chosen at random attended the workshop and showed improvement.

$$P(\text{attended} \cap \text{improved}) = \frac{\text{athletes who attended and improved}}{\text{total number of athletes}} = \frac{44}{100} = 0.44$$

**2** Find the probabilities for the marginal frequencies.

Find the probability that a local player attended the workshop.

$$P(\text{attended}) = \frac{\text{athletes who attended}}{\text{total number of athletes}} = \frac{62}{100} = 0.62$$

Find the probability that a local player improved.

$$P(\text{improved}) = \frac{\text{athletes who improved}}{\text{total number of athletes}} = \frac{50}{100} = 0.5$$

**3** Use the Multiplication Rule to determine if the events are independent or not.

$P(\text{attended})P(\text{improved}) \stackrel{?}{=} P(\text{attended} \cap \text{improved})$

$(0.62)(0.5) \stackrel{?}{=} 0.44$

$0.31 \neq 0.44$

▶ Attending the workshop and improving field goal percentage are not independent events.

**DISCUSS**

Does it make sense that whether or not a player improved would depend on attending the workshop?

# Practice

**For questions 1–4, events *A* and *B* (and *C*, if given) are independent events. Their probabilities are given below. Find the probability requested.**

**1.** $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$

$P(A \cap B) =$ ________________

**2.** $P(A) = 0.7$, $P(B) = 0.3$

$P(A \cap B) =$ ________________

**3.** $P(A) = 0.3$, $P(B) = 0.2$

$P(A \cap B) =$ ________________

**4.** $P(A) = \frac{3}{4}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{2}$

$P(A \cap B \cap C) =$ ________________

**For questions 5 and 6, events *A* and *B* are overlapping events. Their probabilities are given below. Find the probability asked for.**

**5.** $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0.1$

$P(A \cup B) =$ ________________

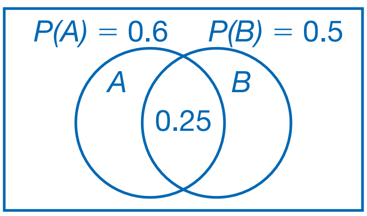
**6.** $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{3}$

$P(A \cup B) =$ ________________

**REMEMBER** $\cup$ is the union of the sets, and $\cap$ is the intersection of the sets.

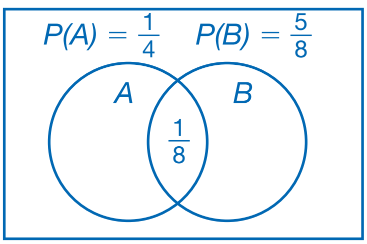
**Find $P(A \cup B)$ given the probabilities in the Venn diagrams.**

**7.**




$P(A \cup B) =$ ________________

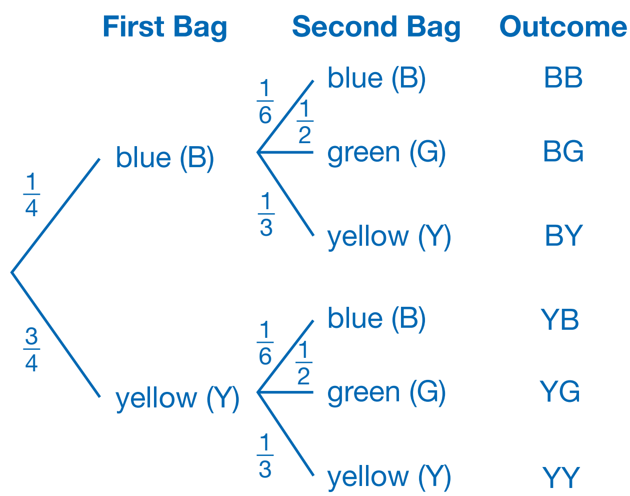
**8.**




$P(A \cup B) =$ ________________

**Use the tree diagram and the information below for questions 9–13.**

Mei-lin has two bags of colored tiles. She will select one tile from the first bag and then select one tile from the second bag. The tree diagram shows the probability for each possible outcome.




**9.** What is the sample space for the experiment? ____________________

**10.** Are all of the outcomes equally likely? How do you know?

______________________________________________________________________

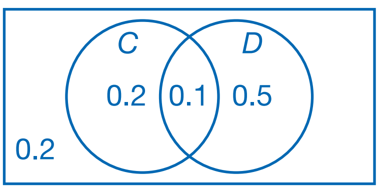
**11.** What is the probability of selecting a blue tile first and a green tile second? ____________

**12.** What is the probability of selecting two yellow tiles? ____________________

**13.** One outcome above has a probability of $\frac{1}{12}$. Which outcome is it? ____________________

**Determine whether or not events *C* and *D* are independent events.**

**14.**



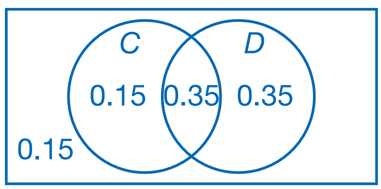

$P(C) =$ ____________ $P(D) =$ ____________

$P(C)P(D) \stackrel{?}{=} P(C \cap D)$

______________________________________

Events *C* and *D* ____________ independent.

**15.**




$P(C) =$ ____________ $P(D) =$ ____________

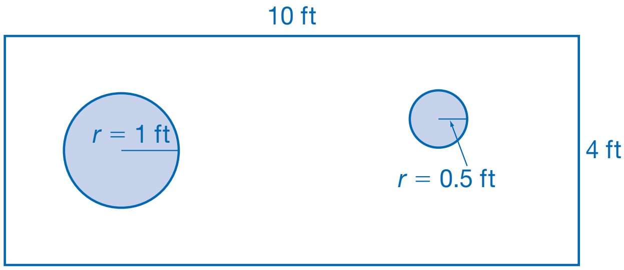
$P(C)P(D) \stackrel{?}{=} P(C \cap D)$

______________________________________

Events *C* and *D* ____________ independent.

**Use the diagram below for questions 16–19.**

A carnival game has players throw table tennis balls into buckets sitting on a platform. If a ball lands in the big bucket (*B*), they win 2 tokens. If a ball lands in the small bucket (*S*), they win 5 tokens. Give your answers as percents rounded to the nearest tenth.




**16.** Find the probability of winning exactly two tokens on a single throw. ____________________

**17.** Find the probability of winning five tokens on a single throw. ____________________

**18.** Are the events *B* and *S* mutually exclusive?

______________________________________________________________________

**19.** Find the probability of winning any tokens on a single throw. ____________________

**Use the two-way frequency table and information below for questions 20 and 21.**

The two-way frequency table shows the results of a survey of 11th-grade and 12th-grade students at Hamilton High School. Students were asked if they do community service.

| | Does Community Service (C) | No Community Service (N) | Total |
|---|---|---|---|
| **11th Grade (E)** | 24 | 12 | 36 |
| **12th Grade (T)** | 16 | 8 | 24 |
| **Total** | 40 | 20 | 60 |

**20.** Suppose an 11th-grade or 12th-grade student at Hamilton High School is selected at random.

What is the probability that the student is either in 11th grade or does community service (or both)?

______________________________________________________________________

What is the probability that the student is either in 12th grade or does community service (or both)?

______________________________________________________________________

21. **SHOW** Are the events "does community service" and "11th grade" independent? Explain your reasoning.

22. **CREATE** Shelley will spin three spinners. The first spinner has two congruent sectors, one red and one blue. The second spinner has four congruent sectors: one red, two blue, and one yellow. The third spinner has nine congruent sectors, three of which are red and the rest of which are blue. Create a tree diagram to represent all the possible outcomes for this experiment. Label the probabilities on each branch. Then determine the probability of spinning blue on all three spinners.